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Products Reliability Prediction Model Based on Bayesian Approach

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***Abstract*— *Predicting reliability of new products at their early life time is one of the important issues in the field of reliability. Lack of data in this period of life time causes prediction to be very hard and inaccurate. This paper proposes a model for predicting non repairable product's reliability early after its production and introduction to the market. It is assumed that time to failure of this product has a Weibull distribution with known shape parameter but the scale parameter is a random variable that could have different distributions like gamma, inverted gamma and truncated normal. Bayesian statistics is used to join prior information on past product failure and sparse few field data on current product's performance to overcome lack of data problem which is a major problem in the early reliability prediction of new products. The Bayesian model provides a more accurate and logical prediction compared to classical methods and indications are favorable regarding the model's practicality in industrial applications. This model has managerial usefulness because of giving more accurate predictions. In all previous studies, there is no comprehensive and precise model for reliability prediction. Different from other studies, we present a definite form for scale parameter of different prior distributions. We use a special form of Weibull distribution which leads us to this definite form. This model provides a suitable estimation value from uncertain environments of parameters because it uses more information for prediction.*

Keywords*—*Reliability prediction, Bayesian theory, Weibull distribution, Prior information.

I. INTRODUCTION

Today's unstoppable growth of industries is very important. From one side increase in competition between industries and from the other side increase in customers demand in terms of reliability of products makes it important for companies to shorten the development time [1], [2]. So, time-to-market and development of product in this sense is one of the major business drivers in the industries [3], [4]. In this case reliability prediction has specific importance. Manufactures also have a greater responsibility for their products' behavior over a longer period, as warranty legislation for their customers. Hence, managers need the quick and reliable prediction method for estimating warranty of their products and also calculating costs of production.

There are different methods for reliability predictions [5]. Prediction based on company's specific fields of data which there may very likely be statistical uncertainty because of a low number of failures. Accelerated life test results are another way to predict reliability of new products which have long life time or when there is a short time between design and production. For this method, there will also be statistical uncertainty but there is additional uncertainty caused by the required extrapolation and the difference between the actual and test environment. Another way for predicting reliability is physics of failure models and they used widely, for example block diagram or fault tree analysis. Publicly available data like RAC are also used for reliability prediction. For RAC data, there may be low statistical uncertainty (because of so many failures) but an overall high degree of uncertainty because operational and environmental stress is unknown. One of the applicable methods in reliability prediction is empirical models like Bellcore, MIL-217. These models generally predict a constant failure rate. Computational modelling techniques is wildly use in many fields such as healthcare [6], education [7], reliability of products [8]. Computer development has increased the usage of computational models in the field of reliability prediction such Neural Networks [9], [8]. Like other fields that machine learning and advanced mathematical methods play an important role for current enhancement [11], in reliably prediction advanced methods play very essential rolls [10].

In the field of reliability prediction models it is suggested to use systematic method for reliability prediction in the design phase, which can help to save money and time. For these complicated models, computer simulation can be useful strategy in order to predicting reliability of systems. Because of the uncertainty in development phase of product's life cycle, using fuzzy logic can be very useful [12], [13]. So, all suitable methods in reliability prediction have been gathered [14].

Lack of data is the main problem arising when trying to predict a product's reliability in the early product's life cycle stage. When not enough data are available, conventional statistics are less useful and provide

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inaccurate predictions. In other words, when sufficient data are not available, using common statistical methods for predicting reliability gives incorrect results [15]. On the other hand, Bayesian approach models probability distributions are based on a limited set of data. So, in this case using Bayesian estimation method gives more acceptable results [16]. There are two main reasons for applying Bayesian methods in reliability problems. The first reason is sparse data and the second one concerns decision theory.

There are only a few studies in the literature review of reliability prediction based on Bayesian approach [17]. It has been developed [18] a Bayesian method for reliability prediction with Weibull distribution for a product's life time. They assume that scale parameter of Weibull has Inverted Gamma distribution and shape parameter has Uniform distribution. Their model contains complex integrals and there is no definite form for reliability distribution. It has been suggested [16] a model based on Bayesian approach and Normal prior for scale and shape parameters of Weibull distribution. This model cannot be accurate since Normal distribution is not an appropriate prior distribution for scale and shape parameter. The reason is that scale and shape parameter in Weibull distribution are positive variables so Normal distribution that covers both negative and positive ranges can't work in this case. It has been suggested [19] a general framework for reliability prediction in the development phase of products based on Bayesian methods. Although their model has many applications, and uses past information and warranty data for prediction, it focuses only on specific Weibull distribution that leads to Exponential distribution. They assumed that the shape parameter is known to change the Weibull distribution to Exponential form and use previous data to fit appropriate Gamma distribution for Exponential parameter. It sounds not to be correct because we have the information for scale parameter of Weibull distribution but do not have it for Exponential. In addition, they study only Gamma distribution as prior for scale parameter. It has also been suggested [20] a model for reliability prediction of product changes through Bayesian inference. As one of the important distributions in reliability topics [21] simplified Weibull distribution with Bayesian approach.

In all previous studies, there is no comprehensive and precise model for reliability prediction. In this paper, we introduce a model which merged limited field data of new product and past information of previous products to have a better prediction. Bayesian statistics are used to join prior information and sparse few field data. Different from other studies, we present a definite form for scale parameter of different prior distributions. We use a special form of Weibull distribution which leads us to this definite form. This model provides a suitable estimation value from uncertain environments of parameters because it uses more information for prediction.

II. MATERIALS AND METHODS

Reliability is generally defined as the probability that a system performs its intended function under operating conditions for a specified period of time. If F(t) is defined as cumulative distribution and f(t) is defined as density distribution of failure rate of one product below equation can be derived

$$R(t) = P(T > t) = 1 - F(t) \tag{1}$$

Reliability prediction means to use statistical models and empirical data to estimate product reliability before real data of new product is available [10]. In this paper we assume that time to failure of the products has Weibull distribution with the following cumulative distribution function;

$$F(t \mid \alpha, \beta) = 1 - e^{-\frac{t^{\beta}}{\alpha}} \tag{2}$$

α is the scale parameter and β is the shape parameter of Weibull distribution. The density function of this distribution is:

$$f(t \mid \alpha, \beta) = \frac{\beta}{\alpha} t^{\beta-1} e^{-\frac{t^{\beta}}{\alpha}} \tag{3}$$

The MTTF of Weibull distribution is given by:

$$MTTF = \alpha^{\frac{1}{\beta}} \Gamma\left(1 + \frac{1}{\beta}\right) \tag{4}$$

In Bayesian models the prior information and the field failure data are used to make the necessary predictions on reliability of the products. The initial input for the process and the prior information are derived from failure analysis on previous generation of the product. In this paper, we assume that shape parameter of Weibull distribution is known and from the previous information, prior distribution for scale parameter will be estimated through the maximum likelihood method.

We purposed different prior distribution for scale parameter, like Exponential, Gamma, Inverted Gamma and truncated Normal. In this paper, we focus more on exponential distribution with the following probability density function we can have

$$g(\alpha) = \frac{1}{\lambda} e^{-\frac{\alpha}{\lambda}} \tag{5}$$

Number of n failure data of product is available. With assumption of having Weibull distribution for this failure data, likelihood function can be written and posterior function based on Bayesian approach can be calculated. With this assumption that β is known, posterior

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distribution for prior gamma distribution can be derived. With respect to a quadratic loss function, Bayesian estimator of α is determined by the following posterior expectation:

$$\hat{\alpha} = \frac{\sqrt{\lambda}\sqrt{\sum_{i=1}^{n} x_i^{\beta}}\, Besselk\left[-2+n, \frac{2\sqrt{\sum_{i=1}^{n} x_i^{\beta}}}{\sqrt{\lambda}}\right]}{Besselk\left[-1+n, \frac{2\sqrt{\sum_{i=1}^{n} x_i^{\beta}}}{\sqrt{\lambda}}\right]} \quad (6)$$

Then new product reliability can be estimated based on following equation

$$\hat{R}(t) = e^{-\frac{t^{\beta}}{\hat{\alpha}}} \quad (7)$$

III. RESULT AND DISCUSSION

Reliability prediction based on past information and few available new data early in product life cycle can become important in two aspects. First from manufacture point of view, because we can reach more accurate reliability prediction of products which is very important for estimating warranty period and pricing. Second reliability prediction is important as consumer point of view. Consumer can calculate life time of new products based on past information to have better decision for maintenance programs. In this study we mention one example from manufacture point of view. Failure data for two previous generations a wireless device is available which time to failure follow Weibull distribution as follow

Fig. 1: Diagram of Reliability for wireless device

Wireless device series I:

$$f_1(t) = \frac{2.7}{4.8\times10^{6}} t^{2.7-1} e^{-\frac{t^{2.7}}{4.8\times10^{6}}}$$

Wireless device series II:

$$f_1(t) = \frac{2.8}{3.6\times10^{6}} t^{2.8-1} e^{-\frac{t^{2.7}}{3.6\times10^{6}}}$$

For wireless device series I, MTTF equal to 261 day and for series II, 439. Now this company produces a new wireless device and wants to increase the MTTF of the new brand. From new wireless devices, 30 samples are taken which failure data for 8 of them is available and the rest are censored. According to past information β=2.85. We assumed that α is a random variable and due to expert opinion has exponential distribution that has λ=1/745 according to pervious data. Likelihood estimation of α parameter based on only new data is 9.1×10^{7}. Parameter α also can be calculated based on only previous data of past product which equals 2.1×10^{7}. If we use Bayesian model for this case we obtain the value of 6.1×10^{7} for α. Table I shows reliability for different values of α.

Table II: Estimation of MTTF and warranty at 85%

	Past Information	New Data	Bayesian Approach
MTTF	367 days	612 days	594 days
Warranty period	287 days	358 days	318 days

The proposed model has many advantages. For example, from manager point of view, (s)he can have an accurate warranty prediction of their new products. As we know, estimating accurate warranty period is very important because it imposes a lot of cost on the company. Assume that the manager assigns number of 85% for warranty period which means 85% of wireless devices don't fail during life cycle. (S)He has two ways to calculate warranty period for his new products. First, he can estimate warranty based on information of past wireless devices. In this case, the warranty periods of about 9 months (287 days) must be expected. Second way is estimating warranty based on only a few data available from new wireless device. Warranty is about 12 months (358 days) based on this data but (s)he must expect more failures after passing a longer time. It is obvious that the result from these two methods can't be logical and accurate, because the first method is based on only past information and the second method is based on only a few data from new product. So, using new method, present in this paper, results in more accurate reliability prediction. For this example the warranty period can be estimated around 10 months (318 days) which is more logical and sounds more accurate. Also, it is very easy and simple to use this model because of definite form for scale parameter that results in definite form for reliability function. So, managers can put the needed parameter that comes from available data to the appropriate model and get the result without solving any complicated integrals.

IV. CONCLUSION

Nowadays, many products are highly reliable and therefore not many failures occur. Even when a large number of them are in use, only limited product failure data may be available. Products are being used under different circumstances, but nevertheless reliability predictions are still based on this sparse and diverse data. Consequently, the predictions are not very reliable themselves. A solution to this problem is the use of Bayesian methods. Bayesian methods combine different data in a uniform way by updating previous information with new data continuously. In other words, Bayes can handle sparse data. In the case of reliability prediction, empirical data for parameter estimation are generally sparse because failures tend to be relatively rare events. Therefore, classical statistical methods, used in these situations, result in wide confidence intervals [5]. In the Bayesian framework, probability distributions are used to express the uncertainties in the parameters.

This paper introduced a Bayesian model for early reliability prediction. A mixture of both past information and new failure data through Bayesian approach showed to provide good response to this problem. So, using more data for predicting reliability is more accurate and the result has more credibility. In this paper, we focus on non-repairable product. This means that we look at a product's first failure. Future repair is not considered, since a second product failure of the same item is unlikely to happen within the warranty period. So it can be suggested for future research to investigate the models for repairable and more complex products. Also, it is recommended to use fuzzy logic for this Bayesian model to solve the uncertainty in parameters of prior distribution.

REFERENCES

[1] P. A. Molenaar, A. J. M. Huijben, D. Bouwhuis, and A. C. Brombacher, “Why do quality and reliability feedback loops not always work in practice: a case study,” *Reliab. Eng. Syst. Saf.*, vol. 75, no. 3, pp. 295–302, Mar. 2002.

[2] M. J. Fard, S. Ameri, S. R. Hejazi, and A. Zeinal Hamadani, “One-unit repairable systems with active and standby redundancy and fuzzy parameters,” *Int. J. Qual. Reliab. Manag.*, vol. 34, no. 3, pp. 446–458, Mar. 2017.

[3] W. Q. Meeker and L. A. Escobar, *Statistical methods for reliability data*. Wiley, 1998.

[4] M. J. Fard, S. Chawla, and C. K. Reddy, “Early-Stage Event Prediction for Longitudinal Data,” in *Pacific-Asia Conference on Knowledge Discovery and Data Mining*, 2016, pp. 139–151.

[5] D. W. Coit, “System Reliability Prediction Prioritization Strategy,” in *Reliability and Maintainability Symposium*, 2000, pp. 175–180.

[6] M. J. Fard, “Computational Modeling Approaches for Task Analysis in Robotic-Assisted Surgery,” Wayne State University, 2016.

[7] S. Ameri, M. J. Fard, R. B. Chinnam, and C. K. Reddy, “Survival Analysis based Framework for Early Prediction of Student Dropouts,” in *Proceedings of the 25th ACM International on Conference on Information and Knowledge Management - CIKM ’16*, 2016, pp. 903–912.

[8] S. Lolas and O. Olatunbosun, “Prediction of vehicle reliability performance using artificial neural networks,” *Expert Syst. Appl.*, vol. 34, no. 4, pp. 2360–2369, May 2008.

[9] L. Tong and Y. Liang, “Forecasting field failure data for repairable systems using neural networks and SARIMA model,” *Int. J. Qual. Reliab. Manag.*, vol. 22, no. 4, pp. 410–420, 2005.

[10] Y.-H. Liang, “Combining neural networks and genetic algorithms for predicting the reliability of repairable systems,” *Int. J. Qual. Reliab. Manag.*, vol. 25, no. 2, pp. 201–210, 2008.

[11] M. J. Fard, A. K. Pandya, R. B. Chinnam, M. D. Klein, and R. D. Ellis, “Distance-based time series classification approach for task recognition with application in surgical robot autonomy,” *International Journal of Medical Robotics and Computer Assisted Surgery*, 2016. DOI: http://doi.wiley.com/10.1002/rcs.1766

[12] M. J. Fard, S. Ameri, and A. Z. Hamadani, “Bayesian Approach for Early Stage Reliability Prediction of Evolutionary Products,” in *International Conference on Operations Excellence and Service Engineering (IEOM)*, 2015, pp. 1–11.

[13] H. Nachtmann and A. Fayetteville, “Fuzzy Reliability in Conceptual Design,” in *Reliability and Maintainability Symposium*, 2003, pp. 360–364.

[14] G. Cooper and G. Thompson, “Concept Design and Reliability,” *Acta Polytech.*, vol. 42, no. 2, 2002.

[15] N. O. Siu and D. L. Kelly, “Bayesian parameter estimation in probabilistic risk assessment,” *Reliab. Eng. Syst. Saf.*, vol. 62, no. 1–2, pp. 89–116, Oct. 1998.

[16] R. A. Ion, H. Kalishoek, D. M. Karydas, and I. Nitescu, “Bayesian Model for Early Reliability Prediction,” in *Reliability and Maintainability Symposium*, 2006, vol. 0, no. C, pp. 260–265.

[17] M. J. Fard, P. Wang, S. Chawla, and C. K. Reddy, “A Bayesian Perspective on Early Stage Event Prediction in Longitudinal Data,” *IEEE Trans. Knowl. Data Eng.*, vol. 28, no. 12, pp. 3126–3139, Dec. 2016.

[18] G. C. Canavos, C. P. Tsokos, and H. Virginia, “Bayesian Estimation of Life Parameters in the Weibull Distribution Bayesian Estimation of Life Parameters in the Weibull Distribution,” *Oper. Res.*, vol. 21, no. 3, pp. 755–763, 1973.

[19] O. P. Yadav, N. Singh, P. S. Goel, and R. Itabashi-Campbell, “A Framework for Reliability Prediction During Product Development Process Incorporating

Engineering Judgments," *Qual. Eng.*, vol. 15, no. 4, pp. 649–662, Jun. 2003.

[20] M. J. H. A. Houben, P. J. M. Sonnemans, G. M. Stollman, P. Healthcare, B. U. Cardio, and M. J. Newby, "Reliability Prediction Through Bayesian Inference Based On Product Change," in *Annual Reliability and Maintainability Symposium*, 2009, pp. 292–297.

[21] A. Aron, R. Corporation, and D. Ogden, "Improving the 1-Parameter Weibull : A Bayesian Approach," in *Annual Reliability and Maintainability Symposium*, 2009, pp. 25–30.